Kinematics & Dynamics of Linkages

Lecture 4: Introduction to Gears

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LAU

Introduction

Why do we need gears?

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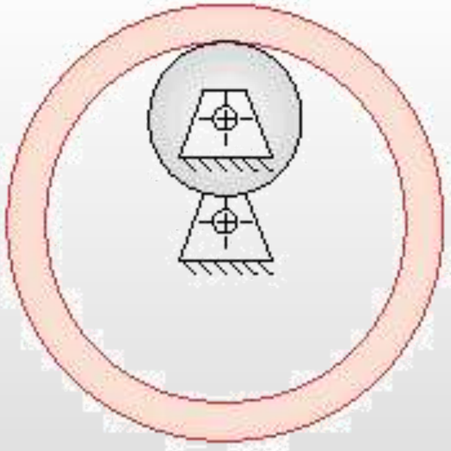
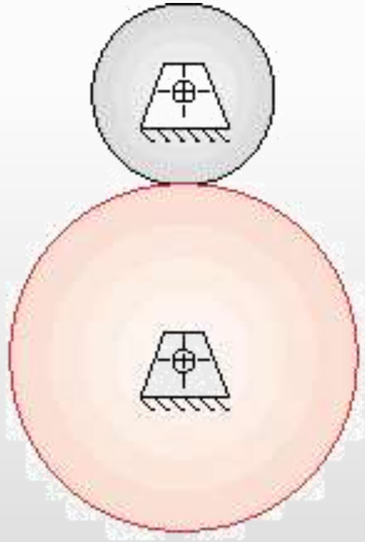
https://s3-us-west-1.amazonaws.com/hydra-lister/Swing_A_Way_407BK_Portable_Can_Opener_Black_Amco_407_0_res.jpg

Introduction – Friction wheels

- The simplest means of transferring rotary motion from one shaft to another is by rolling cylinders with sufficient friction at the interface



Internal set



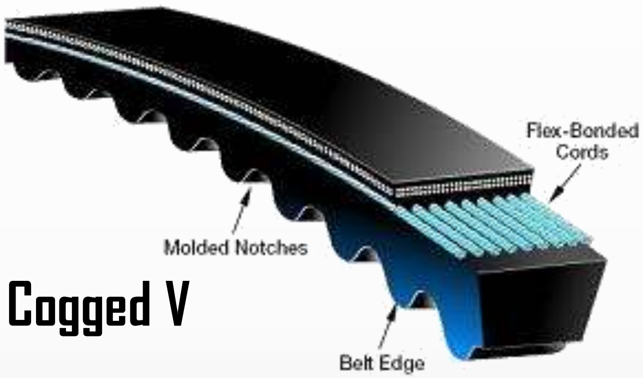
External set

Introduction – Belts

- Flat and V Belts Specs:
    - Transfer power through friction
    - Moderate power applications
    - Enough belt cross-section is needed
    - Possibility of slip
    - Absolute phasing is not possible
    - Quiet and inexpensive solution

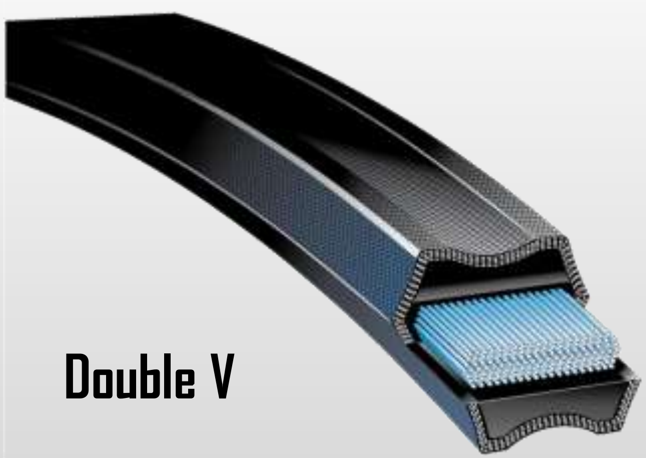
Found in: sewing machines, car alternator

Introduction – Belts



Cogged V

Multiple V

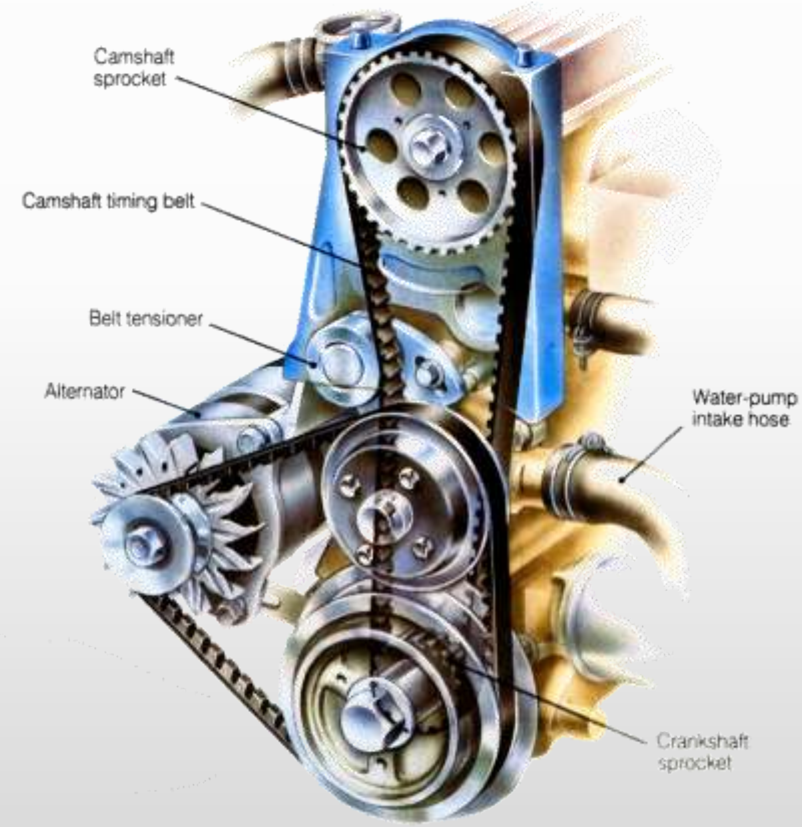


Double V

Standard V

Introduction – Belts

- Timing belts
  - Used for synchronization
  - Transfer power through teeth
  - Lower power applications
  - Absolute phasing is critical
  - Does not slip
  - Also available as double sided timing belts (Serpentine applications)



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Introduction - Gears

- Used when phasing of input & output shafts is required for timing purposes

- **Gear**: larger of two meshing gears

- **Pinion**: small in gear set

Types of Gears

Spur gear

- Most common
- Teeth parallel to axis of gear
- Simple (loud)

Helical gears

- Teeth at a helix angle
- Smoother, quieter (more expensive)
- Types: Parallel or crossed

Types of Gears

Worm gears

- Actually a thread (not an involute)
- Impossible to backdrive (safety)

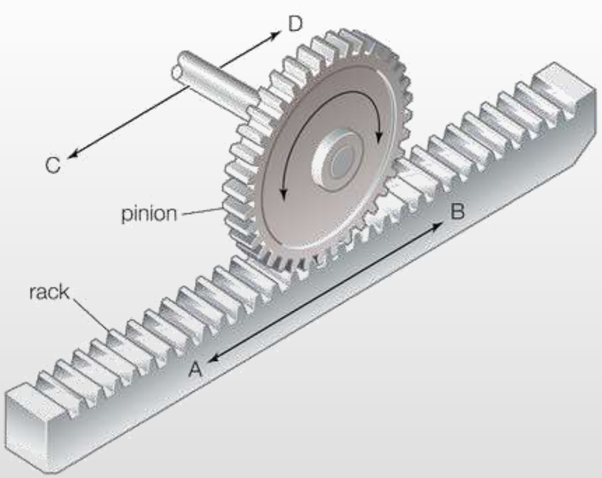
Bevel gears

- Straight : Spur at an angle
- Spiral : Helical at an angle

Types of Gears

Rack and pinion

- Convert rotational motion into linear motion
- Can backdrive



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https://paulsgiganticgarage.com/wp-content/uploads/2017/05/what-happens-when-rack-and-pinion-goes-out.jpg

Gears

Fundamental Law of Gearing

Velocity of A on the pinion: $V_{A_{in}} = r_{in}\,\omega_{in}$

Velocity of A on the gear: $V_{A_{out}} = r_{out}\,\omega_{out}$

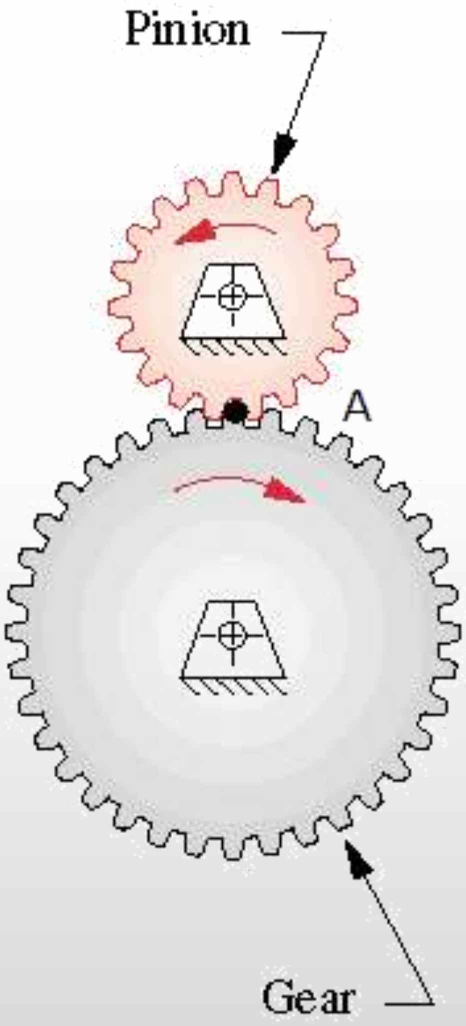
Constraint: $V_{A_{in}} = V_{A_{out}} \quad \rightarrow \quad r_{in}\,\omega_{in} = r_{out}\,\omega_{out}$

Let m_v = angular velocity ratio

Remains constant throughout the mesh

$$m_v = \frac{\omega_{out}}{\omega_{in}} = \pm\frac{r_{in}}{r_{out}} = \pm\frac{d_{in}}{d_{out}}$$

(+ for internal cylinder, – for external cylinder)



Gears

Mating gear teeth contours must be conjugate

- Cycloid tooth found in watches & clocks
- Involute curves in most other gears

Cycloid

Involute

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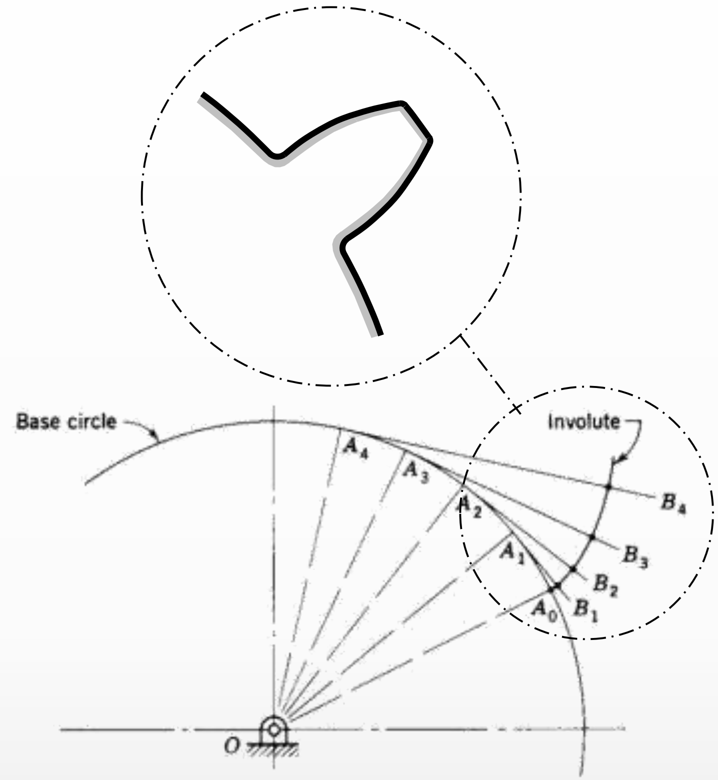
Gears Teeth

Involute curve

Constant pressure angle

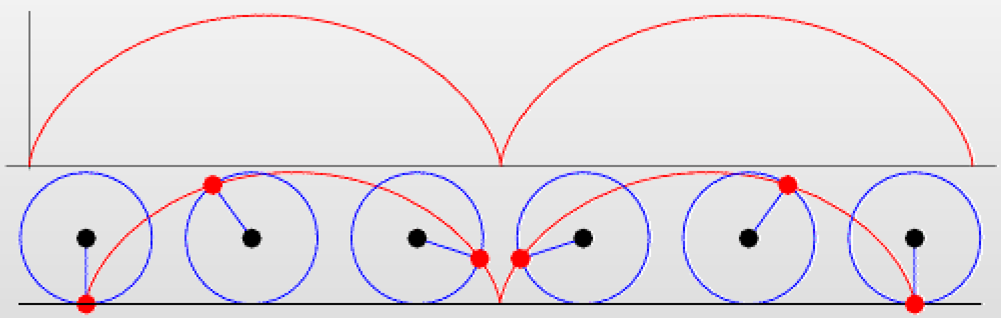
Velocity not affected by vibration



Involute curve

Varying pressure angle

Velocity affected by vibration



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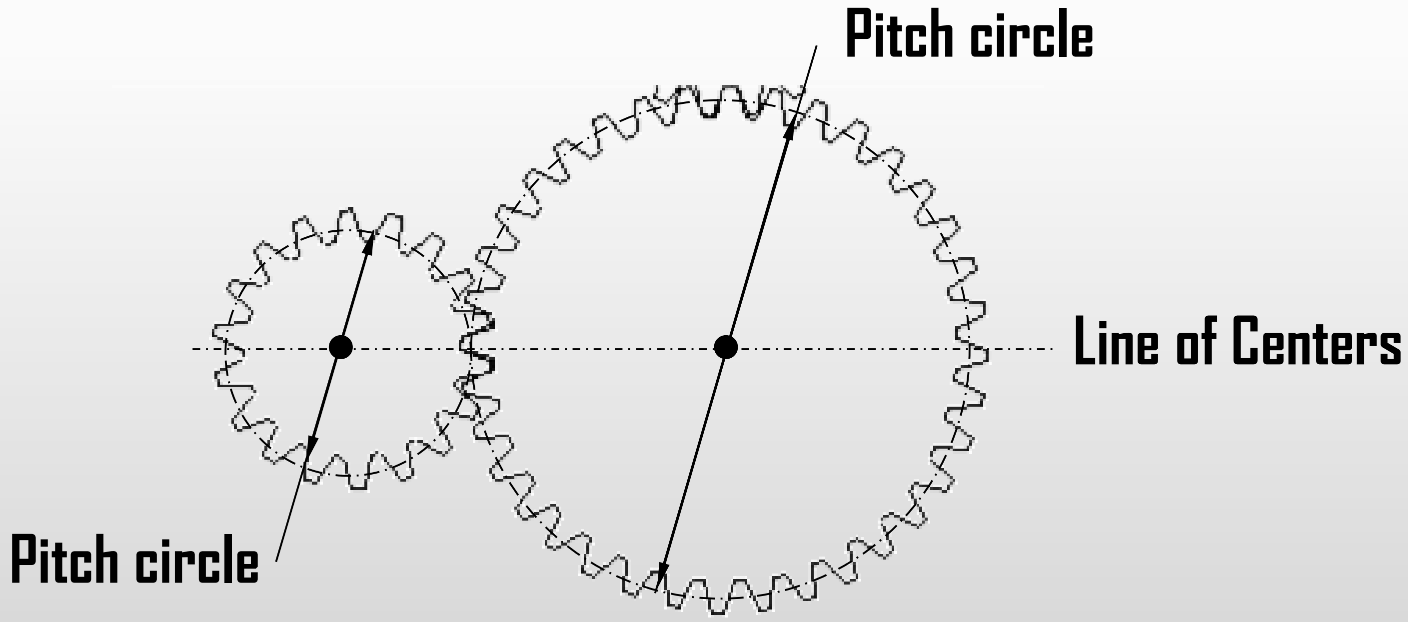
https://cdn-enterprise.discourse.org/mcneel/uploads/default/original/3X/c/b/cbdff2ef68937091c220c0462a9ca89cb64238e8.jpg

Gear Terminology – Main Circles

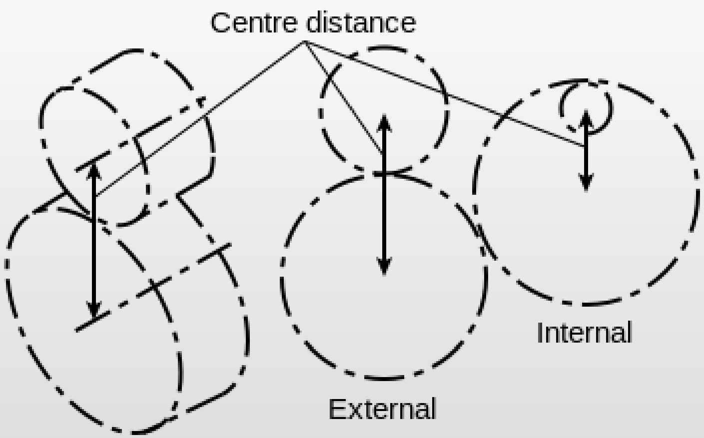
Pitch Circle: Imaginary circle that rolls without slipping with a pitch circle of a mating gear. It is the equivalent circle of friction rollers.

Pitch Diameter: diameter of pitch circle



Gear Terminology – Center distance

The shortest distance between non-intersecting axes. It is measured along the mutual perpendicular to the axes, called the line of centers. It applies to spur gears, parallel axis or crossed axis helical gears, and worm gearing

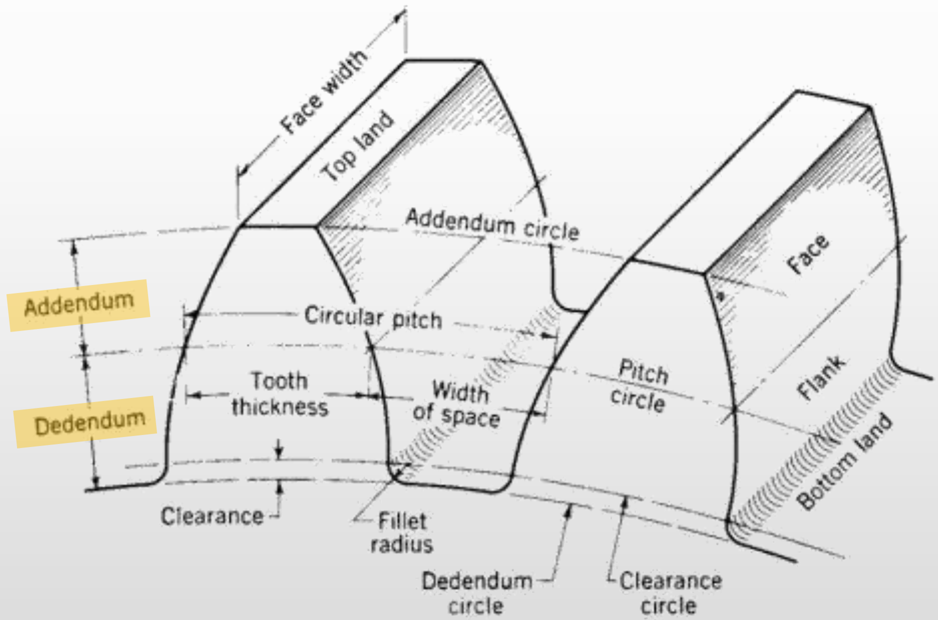


https://en.wikipedia.org/wiki/List_of_gear_nomenclature#/media/File:Center_distance.svg

Gear Terminology – Tooth height

Addendum (a): height of tooth that projects beyond the pitch circle

Dedendum (b): height from bottom of tooth to the pitch circle



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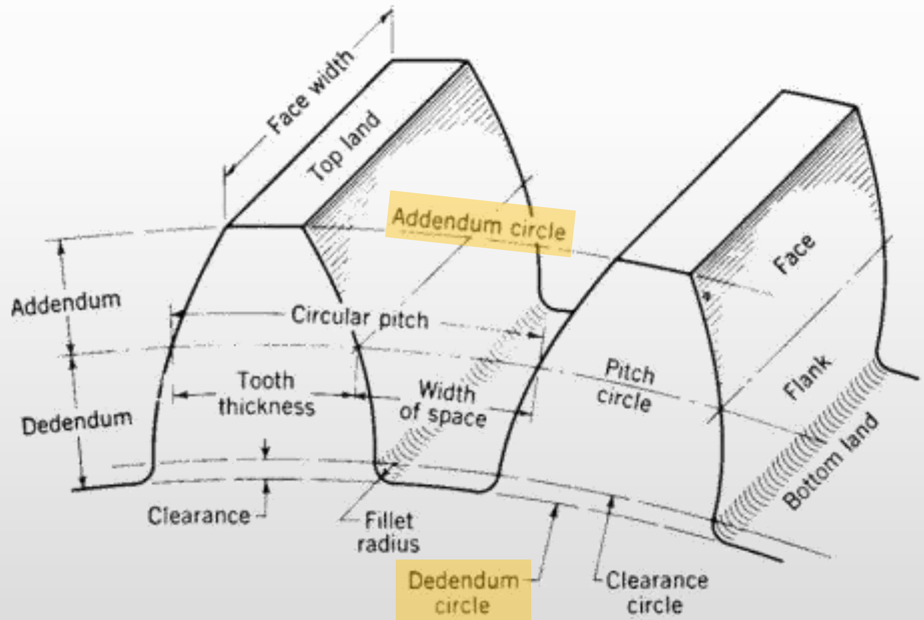
Gear Terminology – Circles

Addendum circle: circle circumscribing the gear

Dedendum circle: circle drawn through the bottom of the gear



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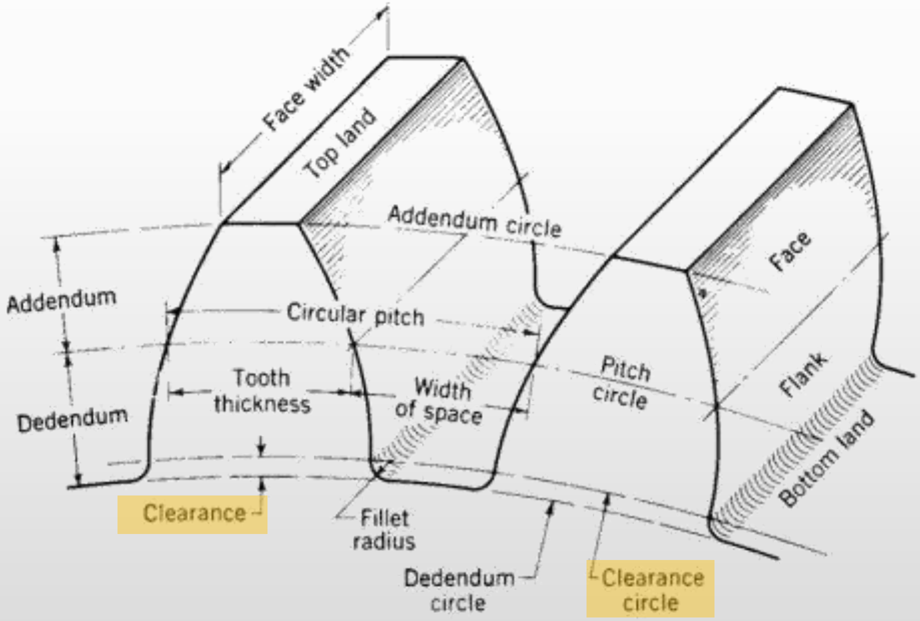
Gear Terminology – Clearance

Clearance circle: largest circle that is not penetrated mating teeth

Clearance: radial distance from the clearance circle to the root circle



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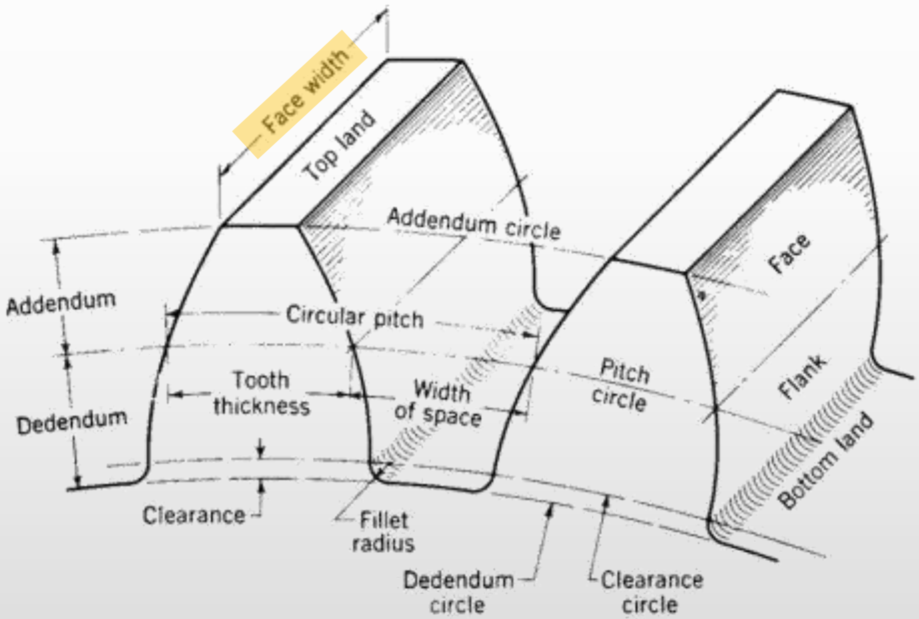
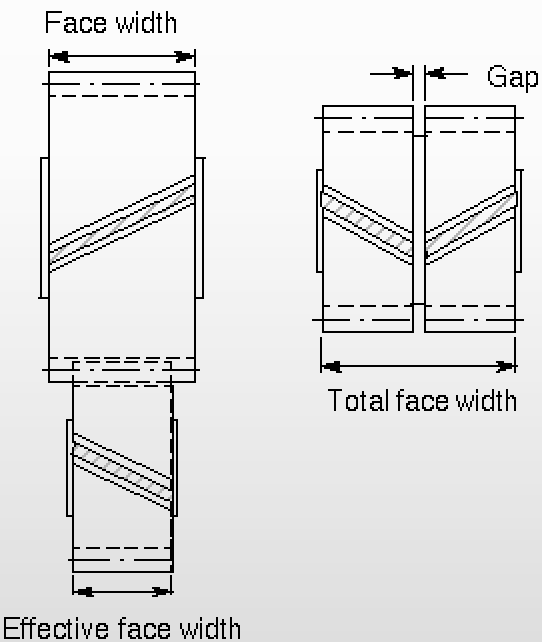
Gear Terminology – Face Width

The **face width** of a gear is the length of teeth in an axial plane. For double helical, it does not include the gap.





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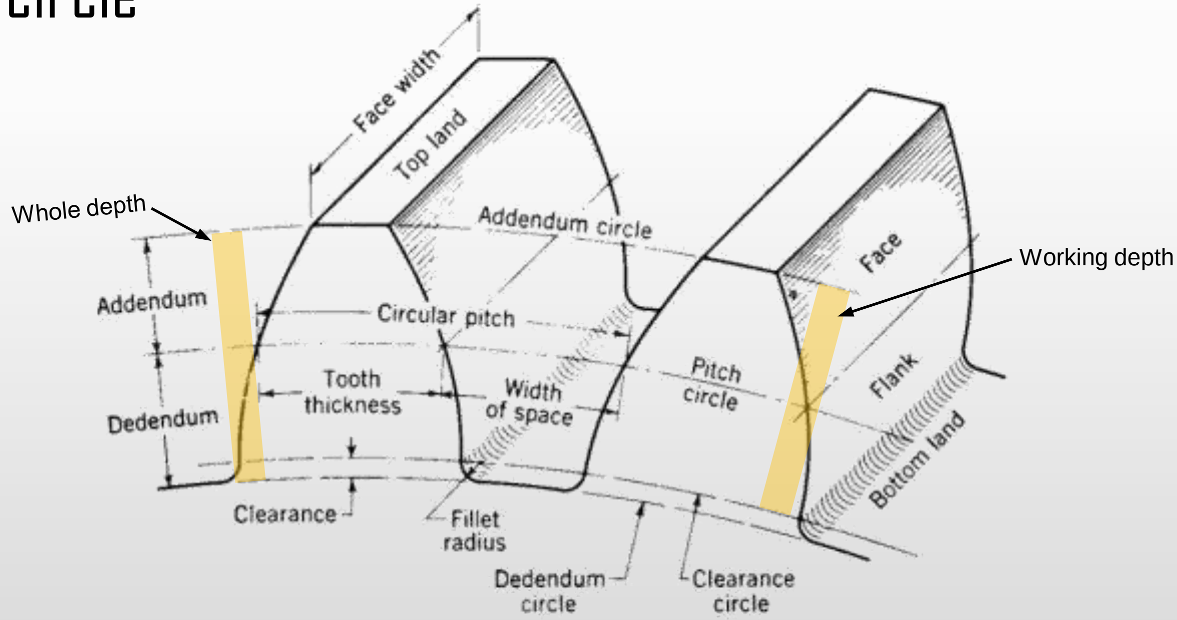
https://upload.wikimedia.org/wikipedia/commons/thumb/6/60/Principal_dimensions.jpg/342px-Principal_dimensions.jpg

Gear Terminology – Depths

Whole depth: distance between the addendum and dedendum circles

Working depth: radial distance between the addendum and the clearance circle



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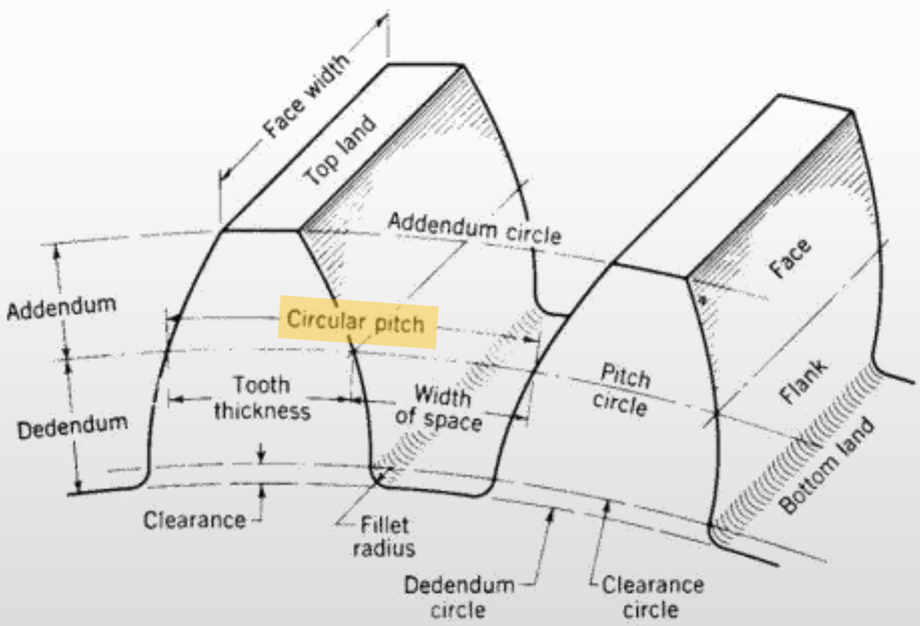
Gear Terminology – Circular pitch

(p): sum of the tooth width and tooth space.
It is equal to the circumference of the pitch circle divided by the teeth.

$$p = \frac{\pi d}{N}$$



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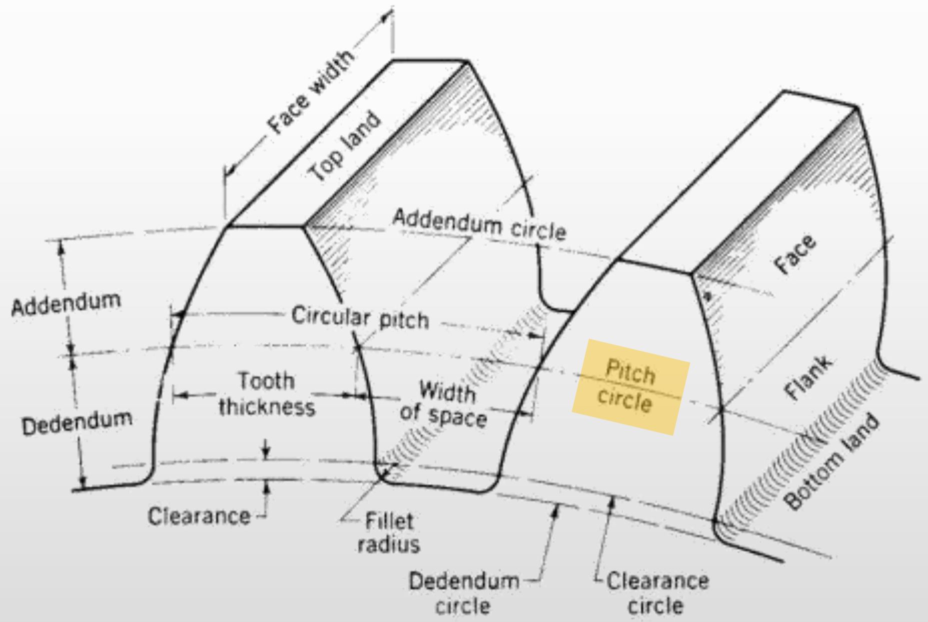
Gear Terminology – Diametral pitch

(P_d): The number of teeth of a gear divided by the diameter of the pitch circle in inches

$$P_d = \frac{N}{d}$$

and

$$P_d\, p = \pi$$



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Diametral pitch

Standard values defined based on available gear cutting tools

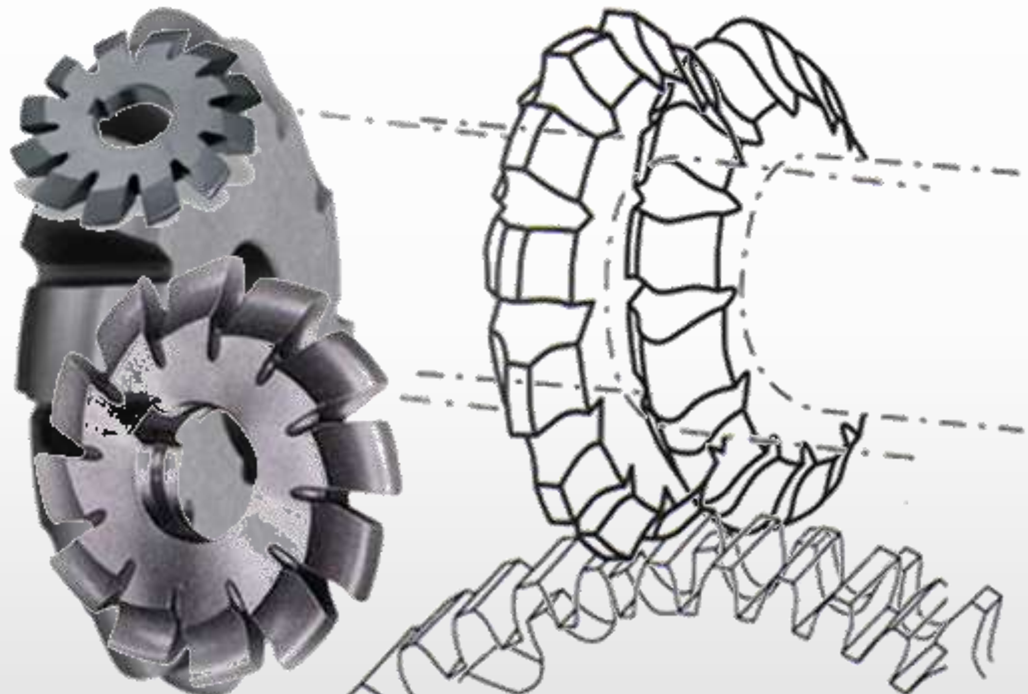
TABLE 9-2
Standard Diametral Pitches

Coarse ($p_d < 20$)	Fine ($p_d \geq 20$)
1	20
1.25	24
1.5	32
1.75	48
2	64
2.5	72
3	80
4	96
5	120
6	
8	
10	
12	
14	
16	
18	

TABLE 9-3
Standard Metric Modules

Metric Module (mm)	Equivalent p_d (in^{-1})
0.3	84.67
0.4	63.50
0.5	50.80
0.8	31.75
1	25.40
1.25	20.32
1.5	16.93
2	12.70
3	8.47
4	6.35
5	5.08
6	4.23
8	3.18
10	2.54
12	2.12
16	1.59
20	1.27
25	1.02

Gear Cutting



https://crtoolsuk.com/wp-content/uploads/2015/05/involute-gear-cutters.jpg

Gear Terminology – Module

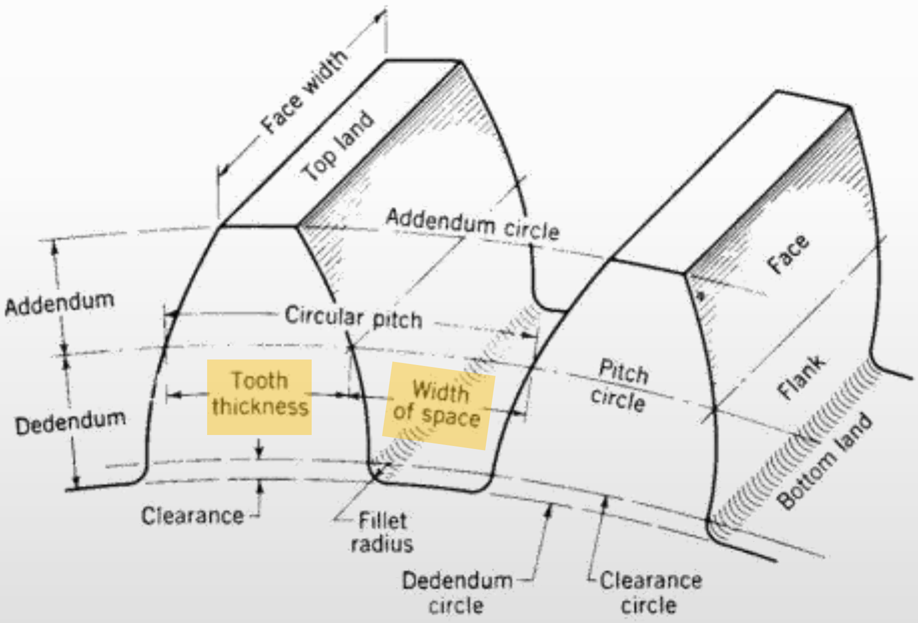
Used in SI units to express the gear tooth size rather than the diametral pitch. It is the reciprocal of the diametral pitch

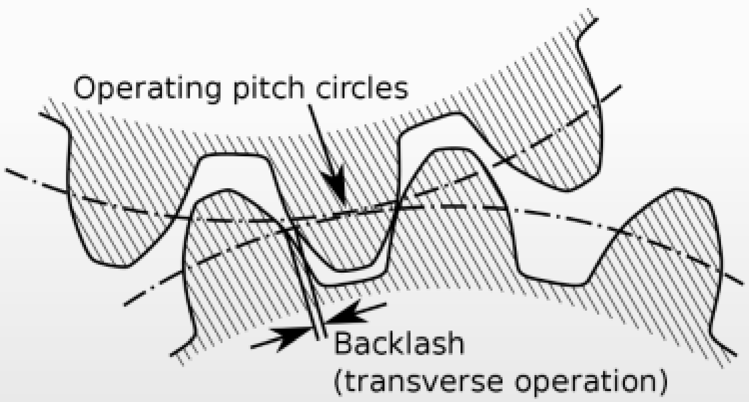
$$m = \frac{d}{N}$$

- Heavy duty Applications need gears with large modules
- Plastic gears with small modules are used in light-duty applications

Gear Terminology – Backlash

The difference between tooth space and tooth thickness.





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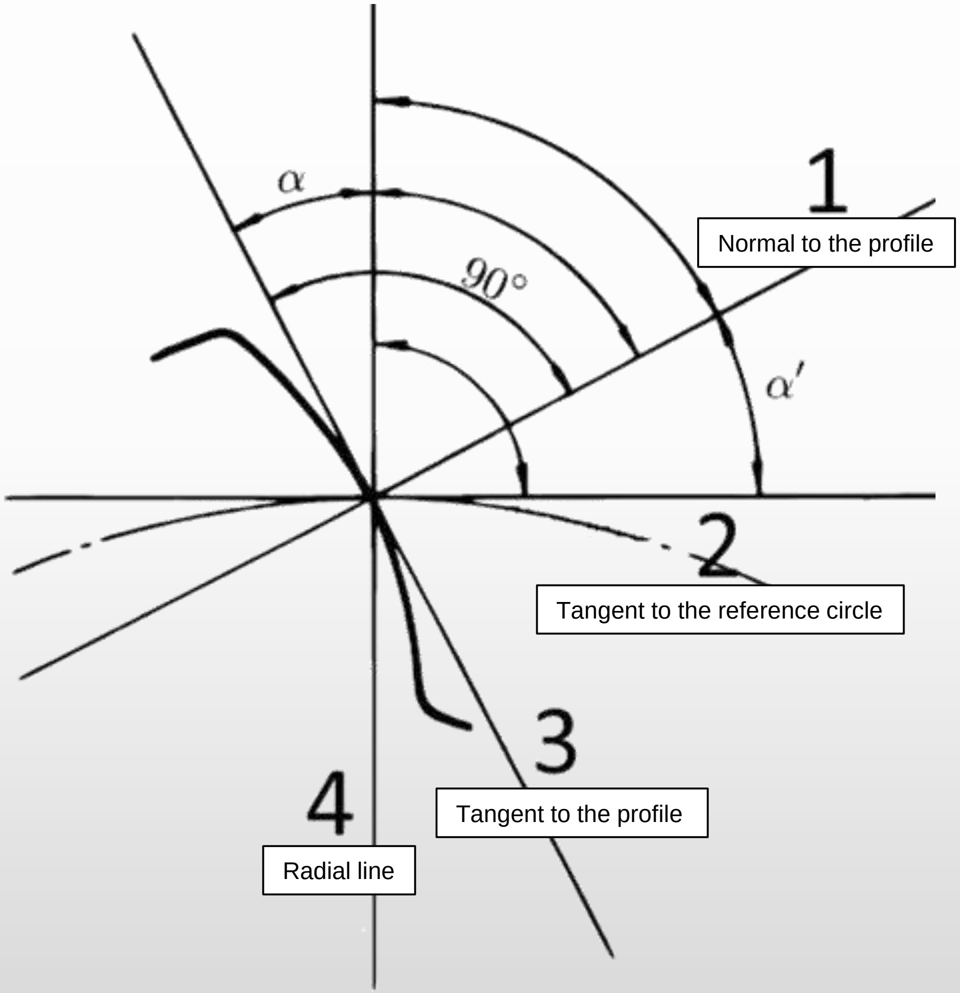
https://upload.wikimedia.org/wikipedia/commons/thumb/6/60/Principal_dimensions.jpg/342px-Principal_dimensions.jpg

Gear Terminology – Pressure Angle

The pressure angle exists between the tooth profile and a radial line to its pitch point. In involute teeth, it is defined as the angle formed by the radial line and the line tangent to the profile at the pitch point.

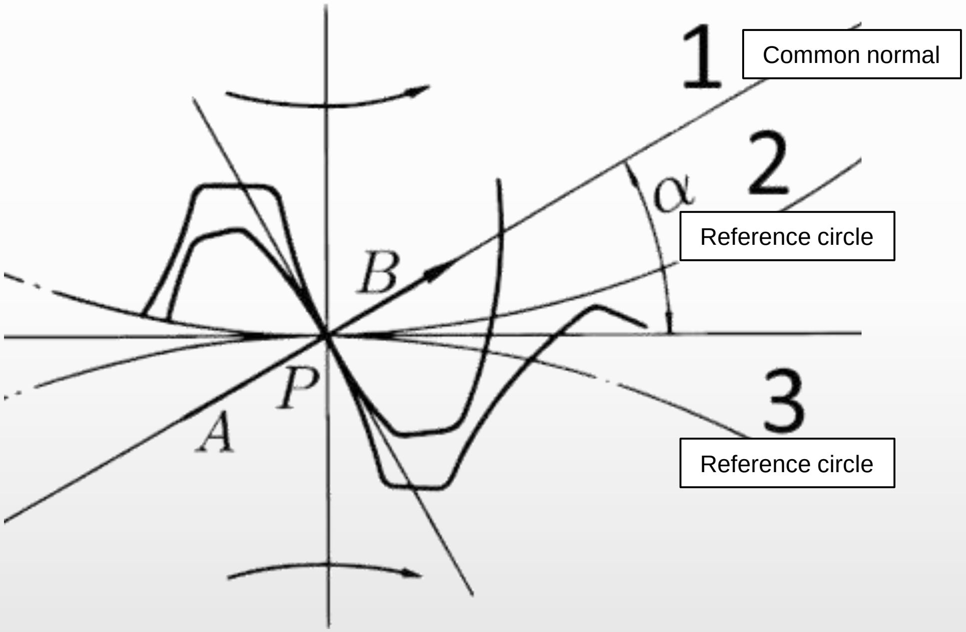


http://khkgears.net/gear-knowledge/introduction-to-gears/pressure-angle/

Gear Terminology – Pressure Angle

At the **pitch point**, the gear **A** is pushing the gear **B**. The pushing force acts along the common normal of the gear A and the gear B.

The pressure angle can be described as the angle between the common normal and the line tangent to the reference circle.



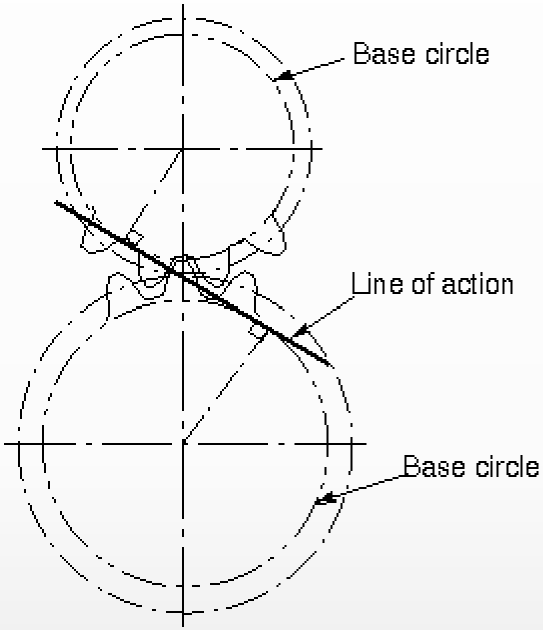
http://khkgears.net/gear-knowledge/introduction-to-gears/pressure-angle/

Gearing Standards – Pressure Angle

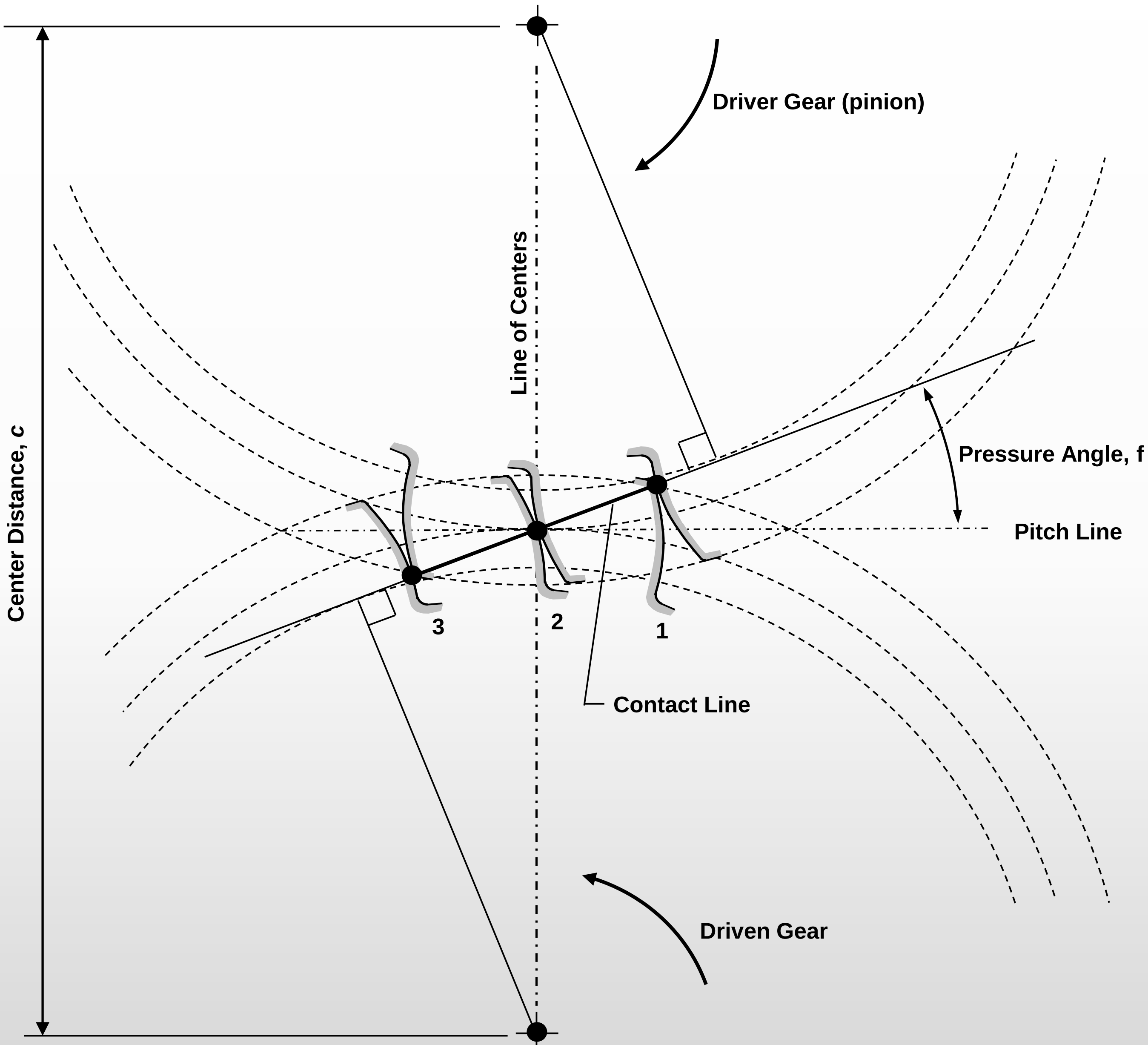
- **14½º** : recommended only for replacement of other 14½º gears
- **20º**: well suited for general applications (most common)
- **25º** : used without concern for interference but have less efficient force transmission, therefore best suited for high speed low-power applications

Gearing Fundamentals

- A pair of meshing gears must have:
  - Same circular pitch
  - Same module
  - Same diametral pitch
  - Same pressure angle



- The shape (profile) of the teeth of a gear must be such that the **common normal** at the point of contact between two teeth always passes through a **fixed point** on the **line of centers** of the gears. This point lies on the pitch circles of both gears



Driver Gear (pinion)
Line of Centers
Center Distance, c
Pressure Angle, f
Pitch Line
3
2
1
Contact Line
Driven Gear

Gear Standard Features

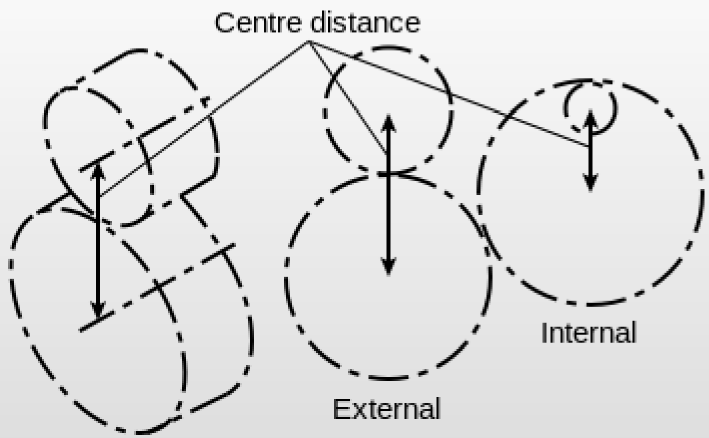
- **Base circle:** $d_b = d \cos\phi$
- **Circular Pitch:** $p = \dfrac{\pi d}{N}$ **Diametral Pitch:** $P_d = \dfrac{N}{d}$
- **Addendum:** $a = \dfrac{1}{P_d}$
- **Dedendum:** $b = \dfrac{1.25}{P_d}$
- **Face Width:** $F = \dfrac{12}{P_d}$

Gear Standard Features

Center distance (c):

$$c_{external} = r_1 + r_2 = \frac{d_1 + d_2}{2} = \frac{N_1 + N_2}{2P_d}$$

$$c_{internal} = r_2 + r_1 = \frac{d_2 - d_1}{2} = \frac{N_2 + N_1}{2P_d}$$



https://en.wikipedia.org/wiki/List_of_gear_nomenclature#/media/File:Center_distance.svg

Gear Standard Features

Contact Ratio (mp) :

Average number of teeth in contact at any instant.

$$m_p = \frac{Z}{p_b}$$

where

Z: length of contact path:

$$Z = \sqrt{(r_2 + a_2)^2 - (r_2 \cos\phi)^2} - r_2 \sin\phi + \sqrt{(r_1 + a_1)^2 - (r_1 \cos\phi)^2} - r_1 \sin\phi$$

and

$\boldsymbol{p_b}$: the base pitch:

$$p_b = \frac{\pi d_1 \cos\phi}{N_1} = \frac{\pi d_2 \cos\phi}{N_2}$$

- Recommend values: 1.4 – 1.5
- Larger values have smoother load transfer

Example 1

A 20°, full-depth, involute spur gear with 18 teeth has a diametral pitch of 12. Determine the following:

- Outside (addendum) diameter.
- Root (dedendum) diameter.
- Standard face width.
- Base circle diameter.
- Circular pitch.

Example 2

Two mating external 4-pitch, spur gears have 14 and 42 teeth. Determine the center distance.

Example 3

Two mating external 4-pitch, 20o , spur gears have 18 and 42 teeth. Determine the contact ratio.

Interference

- If the contact portions of tooth profiles of meshing gears are not involute, then the gears do not execute conjugate action; that is the output gear will not have constant angular velocity. This is called 'interference'.

- Interference happens when:
    - Gears with too few teeth.
    - Small gear mates with a much larger gear.
    - Top of one gear digs into base of the other.

$$N_2 < \frac{\left(N_1{}^2\,(\sin\Phi)^2\ - 4k^2\right)}{4k - 2N_1(\sin\Phi)^2}$$

$$a = \frac{k}{P_d}$$

Interference tables

$\phi = 14½^0$		$\phi = 20^0$		$\phi=25^0$	
Number of pinion teeth	Maximum number of gear teeth	Number of pinion teeth	Maximum number of gear teeth	Number of pinion teech	Maximum number of gear teeth
<23	Interference	<13	Interference	<9	Interference
23	26	13	16	9	13
24	32	14	26	10	32
25	40	15	45	11	249
26	51	16	101	12	∞
27	67	17	1309		
28	92	18	∞		
29	133				
30	219				
31	496				
32	∞				

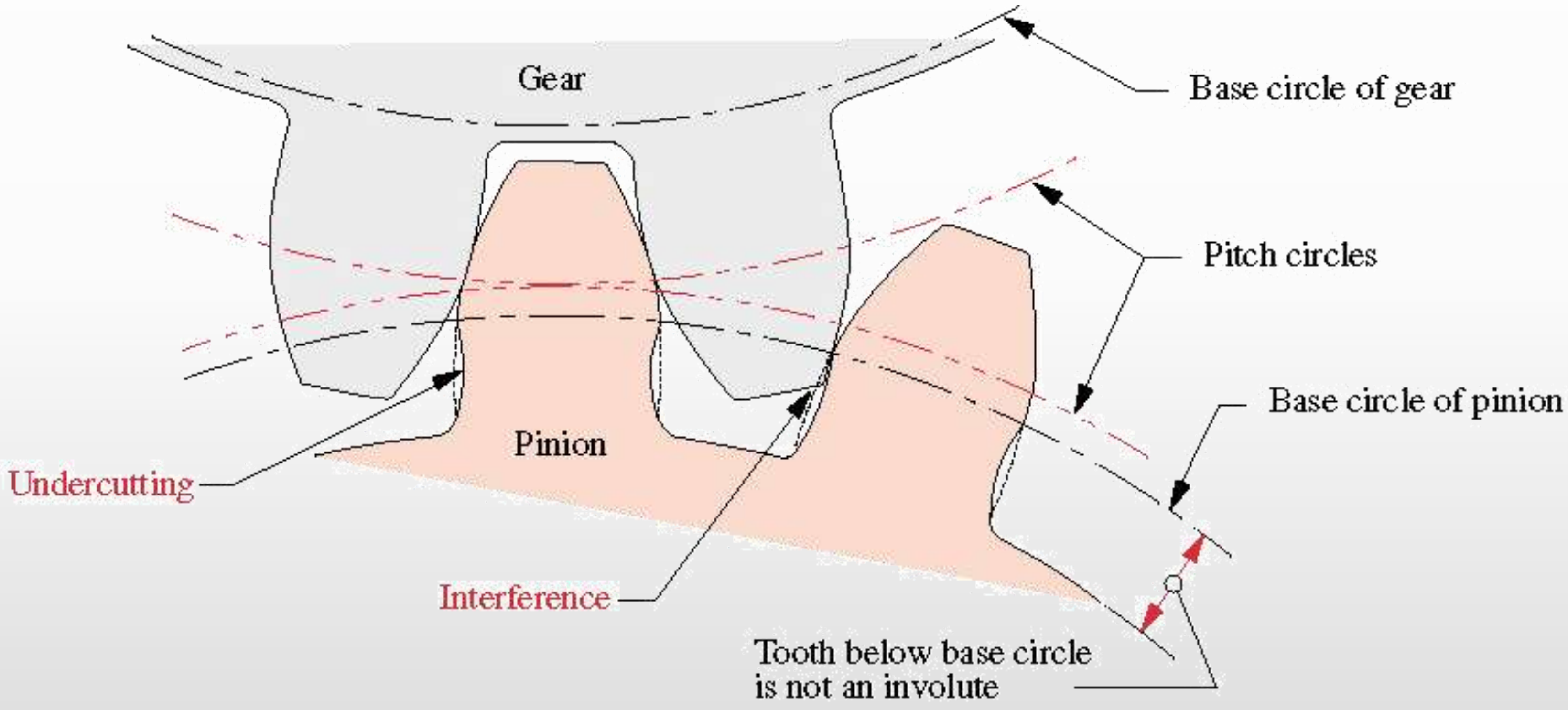
Example 4

Two mating 20°, 4-pitch, spur gears have 12 and 42 teeth.
Will they interfere?

Under cutting

- Under cutting is the removal of material on the gear tooth between the base circle and dedendum circle
- Undercutting reduces the strength of the gear, thus the power that can be safely transmitted.
- Severe undercutting will promote early tooth failure

Under cutting



AGMA Standards

American **G**ear **M**anufacturing **A**ssociation defines standards for quality & tolerances pressure angles numbers of pinion & gear teeth

Table 9–4a	
Min. num. of pinion teeth to avoid interface between pinion and rack	
Pressure angle	Min. num. of teeth
14.5	32
20	18
25	12

Table 9–4b	
Min. num. of pinion teeth to avoid undercutting when cut with a Hob	
Pressure angle	Min. num. of teeth
14.5	37
20	21
25	14

AGMA Standards

Table 9–5a	
Max. num. of Gear teeth to avoid interface (20^0 pinion)	
N. Pinion teeth	Max. Gear teeth
17	1309
16	101
15	45
14	26
13	16

Table 9–5b	
Max. num. of Gear teeth to avoid interface (25^0 pinion)	
N. Pinion teeth	Max. Gear teeth
11	249
10	32
9	13

Gear Selection

Need to decide: Suitable diametral pitch, Pressure Angle, Number of teeth on each gear

TABLE 10.6 Suitable Diametral Pitches for 20°, Mild-Steel Gears with Standard Face Width

Power hp	Pinion rpm 50	100	300	600	900	1200	1800	2400	3600
0.05	20	20	24	32	32	32	32	32	32
0.10	16	20	20	24	24	24	32	32	32
0.25	12	16	20	20	24	24	24	24	24
0.33	10	12	16	20	20	24	24	24	24
0.50	10	12	16	20	20	20	20	24	24
0.75	8	10	12	16	16	20	20	20	20
1.0	6	10	12	16	16	16	20	20	20
1.5	6	8	12	12	16	16	16	16	20
2.0	6	6	10	12	12	12	16	16	16
3.0	5	6	8	10	12	12	12	12	16
5.0	4	5	6	8	10	10	12	12	12
7.5	4	5	6	8	8	8	10	10	10
10	3	4	6	6	6	8	8	8	10
15	2	4	5	6	6	6	6	6	8
20	2	3	4	5	6	6	6	6	-
25	-	3	4	5	5	5	6	5	-
30	-	2	4	4	5	5	5	-	-
40	-	2	3	4	4	-	-	-	-
50	-	-	3	4	4	-	-	-	-

Commercially Available Gears

32 Diametral Pitch

12	16	20	28	36	48	64	80	112
14	18	24	32	40	56	72	96	128

24 Diametral Pitch

12	18	24	30	42	54	72	96	144
15	21	27	36	48	60	84	120	

20 Diametral Pitch

12	16	24	35	50	80	100	160
14	18	25	40	60	84	120	180
15	20	30	45	70	90	140	200

16 Diametral Pitch

12	16	24	32	48	64	96	160
14	18	28	36	56	72	128	192
15	20	30	40	60	80	144	

12 Diametral Pitch

12	15	20	28	42	60	84	120	168
13	16	21	30	48	66	96	132	192
14	18	24	36	54	72	108	144	216

10 Diametral Pitch

12	16	24	30	45	55	80	120	200
14	18	25	35	48	60	90	140	
15	20	28	40	50	70	100	160	

8 Diametral Pitch

12	16	22	32	44	60	80	112
14	18	24	36	48	64	88	120
15	20	28	40	56	72	96	128

6 Diametral Pitch

12	16	24	33	48	66	96
14	18	27	36	54	72	108
15	21	30	42	60	84	120

5 Diametral Pitch

12	16	24	30	45	70	110	160
14	18	25	35	50	80	120	180
15	20	28	40	60	100	140	

Example 5

A gear reducer is used on a concept for a small trolling motor for fishing boats. The gears must transmit **5 hp** from an electric motor at **900 rpm** to the propeller at **320 rpm**. Select a set of gears to accomplish this task.

1. Determine a suitable diametral pitch and pressure angle.
2. Use the required velocity ratio to iterate and determine appropriate number of teeth (confirm that these gears are commercially available).
3. Calculate the pitch diameters and center distance.